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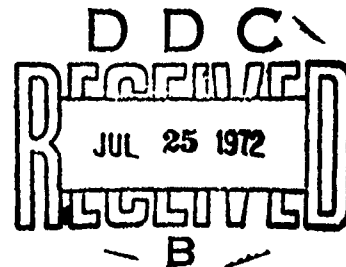
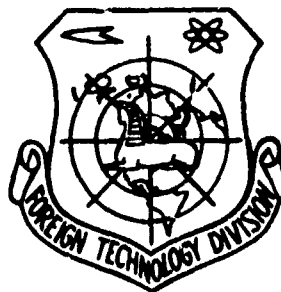
FOREIGN TECHNOLOGY DIVISION



INFLUENCE OF BOUNDARY LAYERS OF ELECTRICAL  
CHARACTERISTICS OF MHD GENERATORS

by

Yu. M. Volkov, D. D. Malyuta  
and V. P. Panchenko



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13. ABSTRACT An analysis is made of the boundary layer effect on the electrical characteristics of a Faraday MHD generator. An electrical substitution circuit and a method for calculating its elements are proposed. An analysis is made of the local circuit for the substitution of a channel in a gas-dynamic approximation. It is noted that the questions of the flow of current through a turbulent boundary layer require careful examination. It has been shown that the existing methods for the calculation of real resistance of a boundary layer, which are based on a break in integration on a certain arbitrary boundary from the electrode, give results which are strongly dependent on the selection of this boundary. Therefore it is advantageous to conduct the performance calculation of an MHD generator for a number of assigned or experimental values of equivalent resistances of the substitution circuit. The real resistance of boundary layers can considerably reduce the output power of a small-scale MHD generators. The boundary layer effect can also become significant for generators of larger dimensions.			

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## INFLUENCE OF BOUNDARY LAYERS OF ELECTRICAL CHARACTERISTICS OF MHD GENERATORS

By: Yu. M. Volkov, D. D. Malyuta and V. P. Panchenko

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PREPARED BY:

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FOREIGN TECHNOLOGY DIVISION  
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# U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

Block	Italic	Transliteration	Block	Italic	Transliteration
А а	<i>А а</i>	A, a	Р р	<i>Р р</i>	R, r
Б б	<i>Б б</i>	B, b	С с	<i>С с</i>	S, s
В в	<i>В в</i>	V, v	Т т	<i>Т т</i>	T, t
Г г	<i>Г г</i>	G, g	У у	<i>У у</i>	U, u
Д д	<i>Д д</i>	D, d	Ф ф	<i>Ф ф</i>	F, f
Е е	<i>Е е</i>	Ye, ye; E, e*	Х х	<i>Х х</i>	Kh, kh
Ж ж	<i>Ж ж</i>	Zh, zh	Ц ц	<i>Ц ц</i>	Ts, ts
З з	<i>З з</i>	Z, z	Ч ч	<i>Ч ч</i>	Ch, ch
И и	<i>И и</i>	I, i	Ш ш	<i>Ш ш</i>	Sh, sh
Й й	<i>Й й</i>	Y, y	Щ щ	<i>Щ щ</i>	Shch, shch
К к	<i>К к</i>	K, k	Ъ ъ	<i>Ъ ъ</i>	"
Л л	<i>Л л</i>	L, l	Ы ы	<i>Ы ы</i>	Y, y
М м	<i>М м</i>	M, m	Ь ь	<i>Ь ь</i>	'
Н н	<i>Н н</i>	N, n	Э э	<i>Э э</i>	E, e
О о	<i>О о</i>	O, o	Ю ю	<i>Ю ю</i>	Yu, yu
П п	<i>П п</i>	P, p	Я я	<i>Я я</i>	Ya, ya

\* ye initially, after vowels, and after ъ, ы; e elsewhere.  
 When written as ѣ in Russian, transliterate as ye or ѣ.  
 The use of diacritical marks is preferred, but such marks  
 may be omitted when expediency dictates.

VI

## ANNOTATION

An analysis is made of the boundary layer effect on the electrical characteristics of a Faraday MHD generator. An electrical substitution circuit and a method for calculating its elements are proposed. An analysis is made of the local circuit for the substitution of a channel in a gas-dynamic approximation. It is noted that the questions of the flow of current through a turbulent boundary layer require careful examination. It has been shown that the existing methods for the calculation of real resistance of a boundary layer, which are based on a break in integration on a certain arbitrary boundary from the electrode, give results which are strongly dependent on the selection of this boundary. Therefore it is advantageous to conduct the performance calculation of an MHD generator for a number of assigned or experimental values of equivalent resistances of the substitution circuit.

The real resistance of boundary layers can considerably reduce the output power of a small-scale MHD generators (for instance, by 2-3 times). The boundary layer effect can also become significant for generators of large dimensions.

The influence of near-electrode resistance on the transverse conductivity in the channel of a Faraday MHD generator when the Hall effect exists is analyzed. The results obtained can be used for the evaluation of the boundary layer effect on the electrical characteristics of an MHD generator as well as for the construction of an algorithm for the numerical computation of flow in the channel.

VII



## INTRODUCTION

The analysis of the experiments conducted up to the present time with MHD generators on combustion products shows that the output characteristics of the generators, especially small-scale, are substantially lower than calculated from the one-dimensional theory [1, 2]. Thus, in a number of experiments the electric power output is 2-3 times less than the computed value [3]. Investigations with the help of probes showed that the distribution of electric field is nonuniform in the interelectrode interval of the MHD channel. It is possible to isolate the flow core and the near-electrode areas of change in potential which occurs at distances of the order of the thickness of the thermal boundary layer on the electrodes. Usually the cathode drop in potential  $\Delta V_c$  in the channel is of the same order as anode drop  $\Delta V_a$ , whereas in the gas discharge  $\Delta V_c \gg \Delta V_a$ . In connection with this it is possible to assume that the potential drop in the channels of generators is specified by the boundary layers on the electrodes and hydrodynamic processes play a substantial role here. This question was discussed in the literature and investigated experimentally [3, 9, 11-16, 19].

In experimental generators a decrease in the open-circuit voltage has also been detected [11, 13, 14]. Since the induced electric field is proportional to the gas velocity, then leakages of current can be expected on the boundary layers on the insulating walls [10, 13].

VIII

Experimentally the facts described above show that surface effects can exert a noticeable influence on the electrical characteristics of generators and on events in the flow core.

Basically this influence amounts to the following.

The cooling of gas in the boundary layer decreases conductivity in the case of an equilibrium molecular plasma and increases the real resistance of the boundary layer. On the other hand, this same effect facilitates the reduction in leakage currents on the boundary layer on the insulating wall.

For relatively short channels whose length comprises several gages the two-dimensional effects become significant, especially when the Hall effect exists, [4, 6]. In this case near-electrode effects decrease the Hall current and create a longitudinal electric field which interferes with effective conductivity in the flow core.

There is another aspect of the influence of surface processes on characteristics of MHD channels. Under conditions of the strong interaction of flow with magnetic field there is the possibility of boundary-layer separation from the electrode wall of the channel. In this case an internal hydro- and electrodynamics of the channel are strongly changed [2, 22].

The influence of near-electrode effects on the hydrodynamics of the channel is not examined here, but primary attention is given to the calculations of near-electrode resistance during the diffuse flow of current through the boundary layer and leakages along the insulating walls. Furthermore an evaluation is made of the influence of near-electrode resistances on the change in the transverse conductivity in the channel of a Faraday MHD generator with continuous electrodes.

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## § 1. THE ANALYZED MODEL OF THE HETEROGENEITIES OF FLOW IN AN MHD CHANNEL

For the typical conditions characteristic for supersonic equilibrium of an MHD generator the Reynolds numbers per unit of length  $Re_x \approx 10^4$ . Since the length of a gas-dynamic circuit (nozzle and channel) comprises several gages ( $\leq 10$ ), then the boundary layers are apparently turbulent, and the channel flow - undeveloped (the boundary layer thickness  $\delta$  less than the characteristic transverse dimension of the channel).

Let us examine the diffuse flow of current in the boundary layer. We will consider that pressure is constant in the cross section of the channel and velocity distribution and temperature correspond to a turbulent boundary layer (a two-layered arrangement of boundary layer was examined). Such an approximation, which subsequently is called gas-dynamic, is fulfilled, if the joule heating and pondermotive forces are small as compared with the heat flux and friction on the wall. In the nucleus of boundary layer at an electric current density  $\sim 1 \text{ A/cm}^2$  and small Hall parameters this approximation is apparently justified. In immediate proximity to the electrode at relatively low wall temperatures it is necessary to consider the processes of diffusion, emission, the energy balance for electrons and heavy particles, and others (see below) [20, 21, 23].

1.1. The velocity distribution in the nucleus of a turbulent boundary layer in the case of exponential approximation takes the form [8]

$$\frac{U}{U_\infty} = \xi^n, \quad (1)$$

where  $\xi = y/\delta$ ;  $y$  - the coordinate, normal to the wall;  $\delta$  - the thickness of the dynamic boundary layer. The exponent  $n$  is the function of the Reynolds number and far from the separation point of the boundary layer takes values in the interval 1/7-1/10. In a viscous (laminar) sublayer the velocity distribution is linear:

$$U = \frac{U_*^2}{\nu(T_w)} y, \quad (2)$$

where  $U_* = U_\infty \sqrt{C_f/2}$  - dynamic velocity;  $C_f$  - the local friction factor;  $\nu(T_w)$  - kinematic viscosity at wall temperature  $T_w$ . The thickness of viscous sublayer  $\delta_\lambda$  and velocity  $U_\lambda$  at  $y = \delta_\lambda$  are determined in the following manner

$$\delta_\lambda = \frac{11.6 \nu(T_w)}{U_\infty \sqrt{C_f/2}}; \quad \frac{U_\lambda}{U_\infty} = 11.6 \sqrt{C_f/2}. \quad (3)$$

1.2. For the boundary layer at the Prandtl number  $Pr \ll 1$  far from the separation point the temperature field is connected with the field of velocities by the following correlation [8]:

$$\frac{\hat{T} - T_w}{T_{aw} - T_w} = \left(\frac{\delta}{\Delta}\right)^{n_T} \cdot \frac{U}{U_\infty} \approx Pr^{0.6} \xi^n, \quad (4)$$

where  $\Delta$  - the thickness of the thermal boundary layer;  $n \approx n_T$ ;  
 $T_{aw} = T_\infty (1 + \gamma \frac{\gamma-1}{2} M_\infty^2)$  - the adiabatic wall temperature;  $\hat{T} = T + \gamma \frac{\gamma-1}{2} T_\infty M_\infty^2 (\frac{U}{U_\infty})^2$ ;  
 $\gamma \approx Pr^{1/3}$  - the recovery factor;  $\gamma = C_p/C_v$  - the adiabatic exponent.

Hence the distribution of static temperature in the boundary layer is expressed thus:

$$T = T_w + (T_{aw} - T_w) Pr^{0.6} \xi^n - \gamma \frac{\gamma-1}{2} M_\infty^2 T_\infty \xi^{2n}. \quad (5)$$

1.3. The transverse conductivity of gas  $\sigma_t$  is determined by scalar conductivity (index "0") and the Hall parameter for electrons  $\beta = R_e \tau_e$ :

$$\sigma_t = \frac{\sigma_0}{1 + \beta^2}. \quad (6)$$

In the boundary layer of a weakly ionized equilibrium gas  $\sigma_0$  and  $\beta$  when pressure gradient does not exist depend only on temperature and in the case of a constant transport cross-section of electron scattering can be presented in the following form:

$$\sigma_0(y) = \sigma_{0\infty} (T/T_\infty)^{3/2} \exp\left[\frac{J_i}{2T_\infty} \left(1 - \frac{T_\infty}{T}\right)\right], \quad (7)$$

$$\beta(y) = \beta_\infty (T/T_\infty)^{1/2}, \quad (8)$$

where  $J_i$  - the potential (energy) of ionization of atoms of the addition.

1.4. The appearance of heterogeneities in the transverse flow pattern (velocity and conductivity) strongly complicate the problem of performance calculation of a generator, which in general becomes three-dimensional. Since the flow is undeveloped it proves to be advisable to separate the phenomena in the flow core and the boundary layer [2, 11, 22], considering, naturally, that the flow core and boundary layer are hydraulically and electrically connected with each other.

Usually the influence of the boundary layer on the hydrodynamics of the channel is considered by the reduction in the through section of the channel by the magnitude of the displacement thickness

$\delta^* = \int_0^{\infty} \left(1 - \frac{u}{u_\infty}\right) dy$  and by the emergence of disturbances in the flow core with boundary-layer separation. With this the development of the boundary layer is determined from the change in parameters in the flow core along the length of the channel calculated without considering the boundary layer effect [22].

To account for the boundary layer effect on the electrical characteristics of a generator various schemes have been suggested [9, 11, 12, 14-16]. As a rule at first the real resistance of the boundary layer is calculated which is then included in the load resistance. This method is apparently valid for channels with subdivided electrodes, when the linear deformation in heterogeneities within the limits of every electrode is small. For the determination of the influence of the heterogeneities of flow on the electrical characteristics of a Faraday generator with continuous electrodes ( $V_g = \text{const.}$ ) under conditions of a powerful change along the length of parameters in the flow core as well as in boundary layer it is expedient to examine the local electrical circuit of substitution, and to calculate the effective characteristics of the channel by means of series integration along the length. The introduction of the local circuit of substitution makes it possible to take into account the influence of the heterogeneities of flow on parameters in its nucleus, therefore on flow as a whole.

The joint calculation of the flow core and boundary layer should be conducted in series and is possible in practice only when using a computer, whereupon the correlation between the nucleus and boundary layer is considered on every step of the calculation network.

The calculation of flow in the flow core is reduced to the solution of a Cauchy problem for a system of ordinary differential equations. The number of initial conditions include, for example, the load coefficients for the nucleus  $K_\infty(K_i)$  or voltage on the boundaries of the flow core or electrodes, and also all the boundary layer characteristics. As a result of numerical solution the values are found for all variables in the following node of the calculated grid, besides  $K_\infty$  and the values connected with  $K_\infty$ , and also the characteristics necessary for the boundary-layer calculation ( $\frac{\partial^2}{\partial x^2}$ ,  $\frac{d\rho_\infty}{dx}$ , form-parameters  $f = \frac{\delta^{**}}{u_\infty} \frac{d u_\infty}{dx}$ ,  $H = \frac{\delta^{**}}{\delta^{**}}$  and others). Further the numerical solution of the boundary layer equations is carried out for the electrode and insulating walls. This determination of the boundary layer characteristics makes it possible to calculate (for instance, by the iterative

method) the load coefficient  $K_{\infty}$  which satisfies the condition of constancy of electrode voltage  $V_s = V_{s,i} = \text{const}$ . The complexity of computation of  $K_{\infty}$  is connected with the fact that the near-electrode drop in voltage in general is the function of current density and therefore the load coefficient.

After the agreement of electrode voltage the calculation procedure is repeated.

Below the local circuit of substitution is analyzed and the evaluations of the magnitudes of its individual elements are given.

## § 2. LOCAL ELECTRICAL CIRCUIT FOR SUBSTITUTION OF AN MHD CHANNEL

2.1. Let us examine the stationary flow of a viscous compressed gas ( $P_r \leq 1$ ) with equilibrium electrical conductivity in the rectangular channel of a Faraday MHD generator at small magnetic Reynolds numbers. We will consider that flow in the flow core is one-dimensional, end effects do not influence the picture of current distribution in the section in question, boundary layers are flat and their thicknesses on the anode and cathode are equal, and the induction of magnetic field is constant in the section of channel.

Figure 1 shows the picture of the current distribution in question in the cross-sectional flow of the channel, and Fig. 2 - the distribution of velocity and temperature in the boundary layers. For the engineering calculation of electrical characteristics of the MHD channel with a flow which is heterogeneous in cross section the local electrical circuit of substitution was used. The precise calculation of the distribution of fields and currents in the cross section of the channel is complex and requires the solutions of a two-dimensional boundary value problem [5, 7].

Figure 3 depicts the proposed local circuit of substitution, qualitatively corresponding to the picture of current distribution which was shown in Fig. 1 (subsequently all values are related to a unit length of channel, for example, current  $J \equiv dI/dx$ , resistance  $R' \equiv \frac{d\tau}{dx}$ , where  $I$ ,  $\tau$  - the integral values of current and resistance).



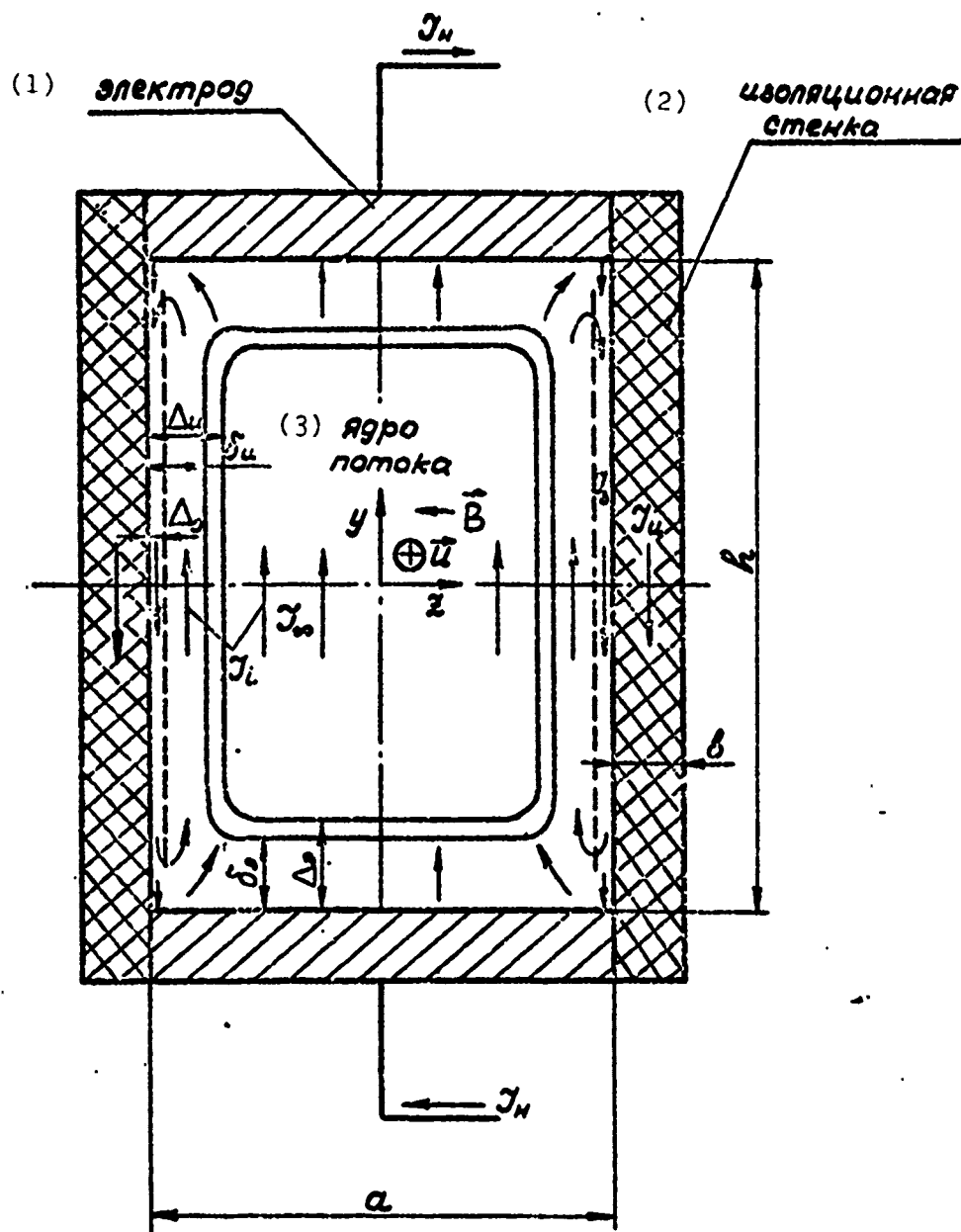


Fig. 1. Cross section of MHD generator channel.  
KEY: (1) electrode; (2) insulation wall; (3) flow core.

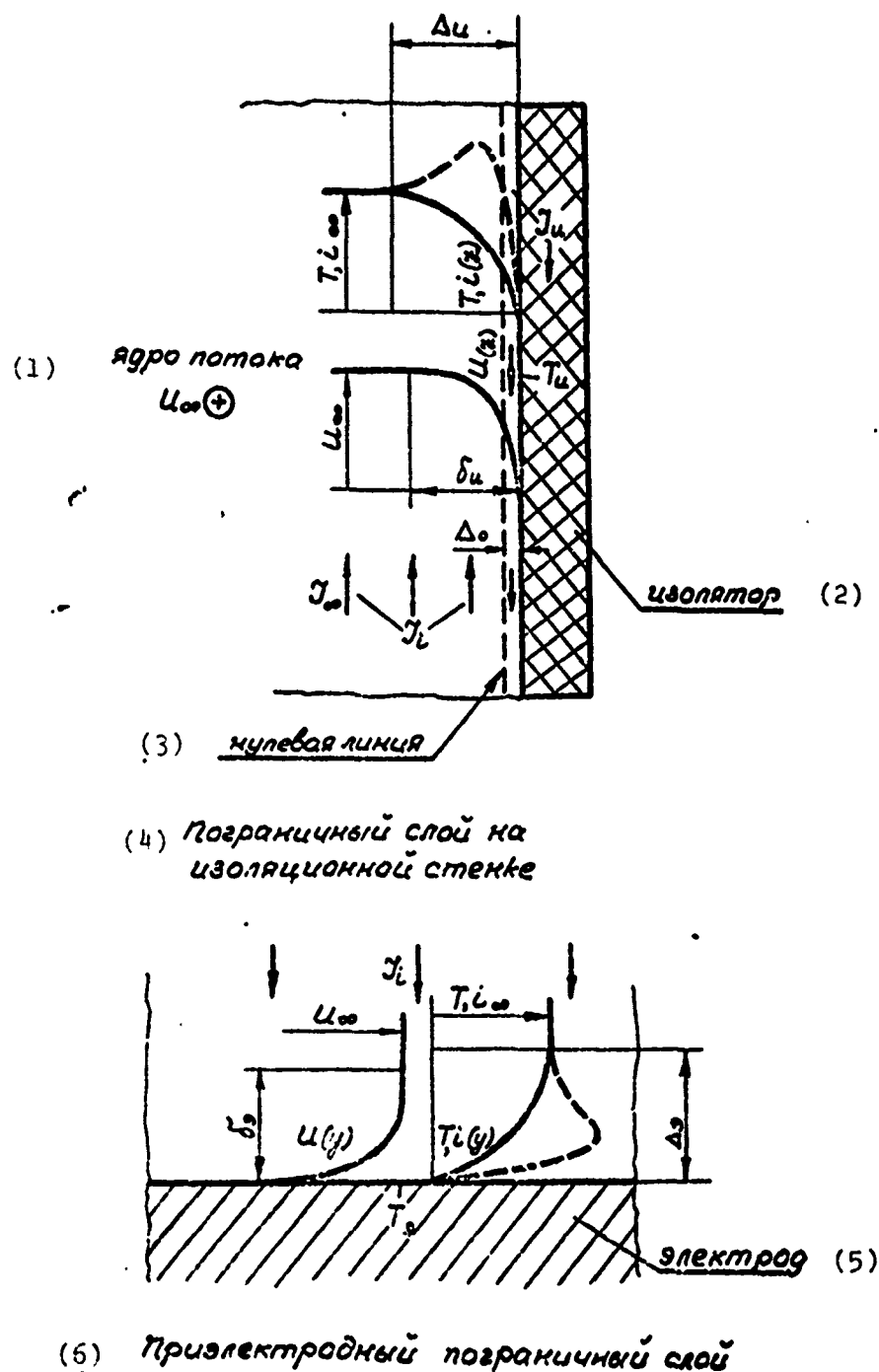


Fig. 2. Boundary layers on insulating and electrode walls.  
KEY: (1) flow core; (2) insulator; (3) zero line; (4) boundary layer on insulating wall; (5) electrode; (6) near-electrode boundary layer.

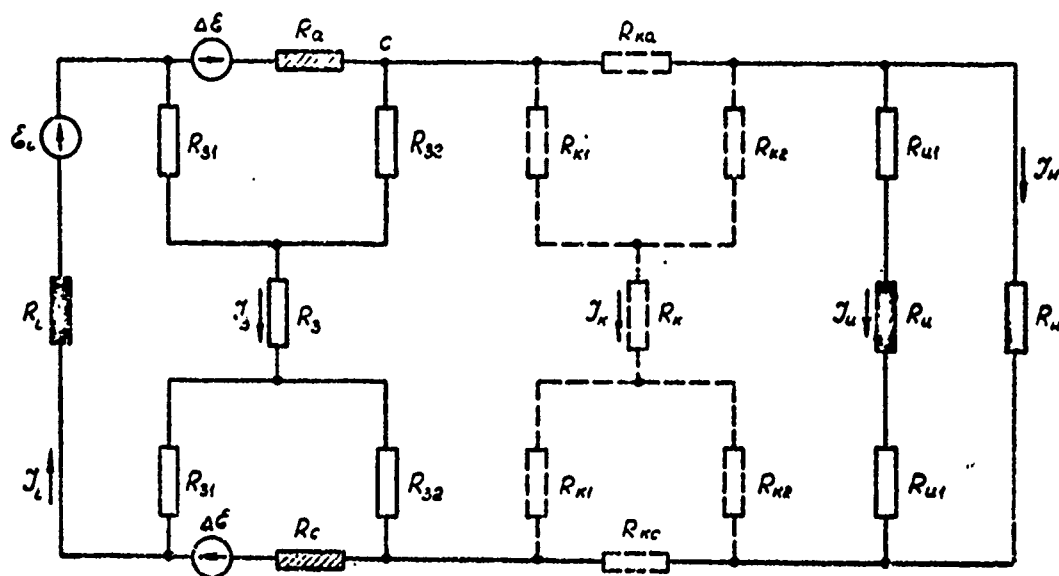


Fig. 3. The electrical circuit of substitution.

In Figs. 1-3 the following designations are introduced:  $\Delta_3$  - the thickness of the thermal boundary layer on the electrode;  $h$ ,  $a$  - interelectrode distance and width of through section of the channel;  $\Delta_0$  - zero line (see below);  $\mathcal{E}_i \equiv \mathcal{E}_\infty = u_\infty B(h - 2\delta_3)$  - emf generated in the flow core;  $\Delta \mathcal{E} = \int_0^{\delta_3} u B dy$  - emf generated in the boundary layer on the electrode. We will assume that in the area  $\sim (-h/2 + \Delta_3) - (h/2 - \Delta_3)$  the equipotentials are parallel to the plane of the electrode over the entire channel width, and the leakages of current from the flow core can be significant only near the angle of the channel under conditions of operation of the generator close to no-load running.

Such an assumption makes it possible to introduce the local load coefficient for the flow core, determined analogously with the one-dimensional theory:

$$K_i \equiv K_\infty = E_i / u_\infty B, \quad (9)$$

The circuit of substitution (Fig. 3) includes the resistors of the

flow core  $R_i$ , load  $R_n$ , near-electrode resistors  $R_a$ ,  $R_c$  and three banks of resistors which shunt the load:  $R_s$ ,  $R_k$ ,  $R_u$ . The resistors of group  $R_{sj}$  determine the leakage of current  $J_s$  over the boundary layers on the insulation walls. Effective resistance  $R_s \approx V_s/J_s$ , where  $V_s$  - voltage on the section from  $(-a/2 + \Delta_0)$  to  $(-1/2 - \Delta_0)$ ;  $R_{s1}$  determines the fraction of leakage current which does not reach the electrode, and  $R_{s2}$  - the fraction of leakage current from the electrode into the area  $a/2 - (a/2 - \Delta_0)$ .

For some working conditions on the internal surface of channel a layer of electro-conductive material can be formed (deposition from the flow, products of the reaction of the material of the electrodes with the gas, etc.). The influence of such a layer on the electrical characteristics of a generator can be formally taken into account by means of the introduction into the circuit of substitution of the bank of resistors  $R_{sj}$ , each of which is analogous to the corresponding resistor of group  $R_{sj}$ . Resistance  $R_u$  is the resistance of leakage current  $J_u$  on the insulating walls of the channel. Let us note that resistances  $R_a$ ,  $R_c$ ,  $R_{s2}$  cannot be determined by the boundary layer and by contact resistance, but by the resistance of micro-arcs which appear in a number of cases at large electric current densities. These resistances can depend on current and magnetic field considerably more strongly than in the case of the diffuse flow of a current [18]. This case is not examined here.

2.2. The line in the boundary layer on insulating wall far from the electrode, where the electric current density is equal to zero, let us call the zero line. Condition  $J_y = 0$  corresponds to equality  $E_i = u(\Delta_0)B$ . Hence the relative distance of the zero line from the wall is determined thus:

$$\frac{\Delta_0}{\delta_u} = K_i^{1/n}. \quad (10)$$

Since the strength of current leakage on boundary layer is proportional to  $\Delta_0$ , then it can be significant only at  $K_i \sim 1$ , and  $\Delta_0$  exceeds the thickness of the laminar sublayer  $\delta_\lambda$  ( $\delta_\lambda < 0.1\delta$ ) only at  $K_i \geq 0.8$ .

At  $\Delta_0 < \delta_\lambda$  the value of  $\Delta_0$  is linear on  $K_i$ :

$$\frac{\Delta_0}{\delta_\lambda} = K_i \left( \frac{\delta_\lambda}{\delta_u} \right)^{-n} \quad (11)$$

In the mode of free running ( $R_M \rightarrow \infty$ ) on any straight line  $y = \text{const}$  should satisfy the condition:

$$\int_0^{a/2} j_y(z) dz = 0 \quad \text{or} \quad |J_i| = |J_s|, \quad (12)$$

i.e. always  $K_i < 1$ .

The density of transverse current is determined by the Ohm law:

$$j_y(z) = \sigma_z(z) [u(z)B - E_y(z)]. \quad (13)$$

For the evaluation of the open-circuit voltage we will assume that

$$\int_0^{a/2} \sigma_z E_y dz = \tilde{E}_y \int_0^{a/2} \sigma_z dz.$$

In this case the open-circuit voltage is

$$V_{xx} = \int_{-1/2}^{1/2} \frac{\int_0^{a/2} \sigma_z E_y dz}{\int_0^{a/2} \sigma_z dz} dy. \quad (14)$$

If conductivity is constant in the cross section of the channel, then open-circuit voltage, with other conditions being equal, is determined by the velocity profile. The decrease in conductivity in the boundary layer can lead only to an increase in the open-circuit voltage as compared with the case  $\sigma = \text{const}$ .

In the case of the exponential approximation of the velocity profile with index  $n$  the expression for the open-circuit voltage can be written as (for simplicity it is considered that  $\delta = \delta_s = \delta_u$ ):

$$V_{xx} = u_{\infty} B h \left\{ \left( 1 - \frac{2\delta}{\alpha} \frac{n}{n+1} \right) \left( 1 - \frac{2\delta}{h} \right) + \frac{2\delta}{h} \frac{1}{n+1} \left( 1 - \frac{n}{n+2} \frac{2\delta}{\alpha} \right) \right\}. \quad (15)$$

Figure 4 shows the change in the dimensionless open-circuit voltage  $\bar{V}_{xx} = V_{xx}/u_{\infty} B h$  depending on the relative boundary layer thickness for  $n = 1/2 - 1/10$  at  $\alpha = h$ .

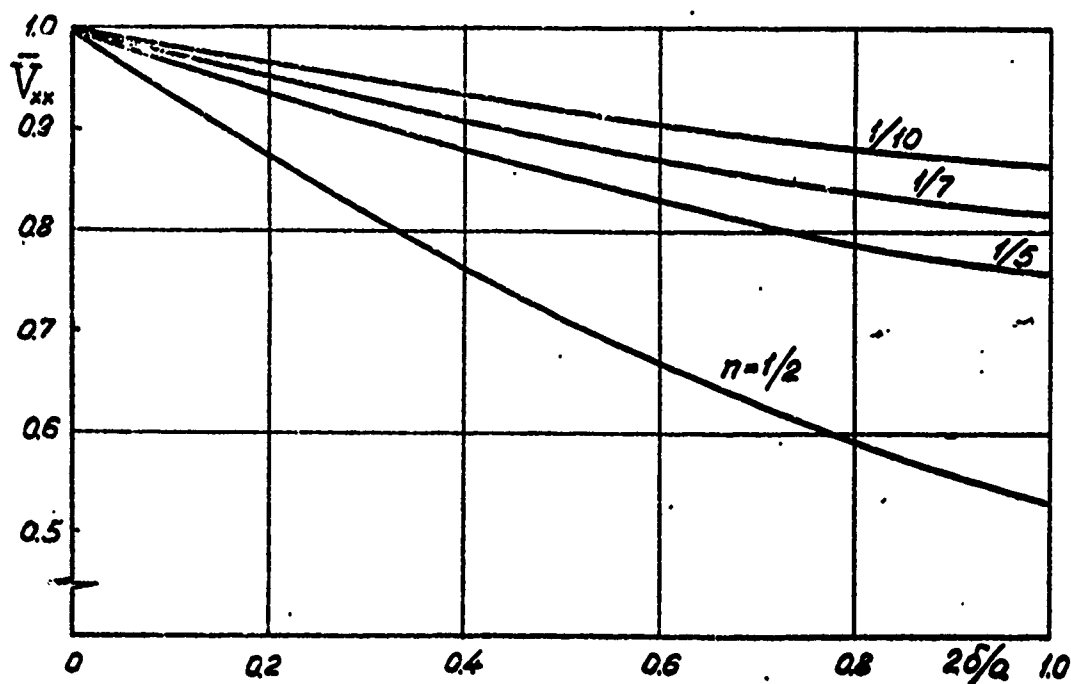


Fig. 4. The dimensionless open-circuit voltage  $\bar{V}_{xx} = V_{xx}/u_{\infty} B h$  depending on the relative boundary-layer thickness  $2\delta/\alpha$  ( $\sigma_1 = \text{const}$ ,  $\alpha = h$ ,  $u \sim x^n$ ).

It is evident that at  $n \leq 1/7$  and  $2\delta/\alpha \leq 0.5$  the reduction of  $V_{xx}$  due to the leakages of current over the boundary layers on insulating walls is less or of an order of 15%. Therefore during the engineering calculation of the nominal mode of operation of a generator ( $K_L \leq 0.8$ ) the leakages of current in the turbulent boundary layers can be disregarded.

2.3. Current on the electrode in the examined model can be calculated in the following manner:<sup>1</sup>

$$J = 2 \int_0^{a/2} j_y(z) dz, \quad (16)$$

where  $j_y(z) = \sigma_i(z) U_{\infty} B \left[ \frac{U(z)}{U_{\infty}} - K_i \right] = \sigma_i(z) [U(z) B - E_i].$  (17)

Disregarding the leakages of current in the nominal mode of operation the current generated in area  $(\frac{h}{2} + \Delta_s) \div (\frac{h}{2} - \Delta_s)$  is determined in the following manner ( $P_e \approx 1$ ):

$$J_i \approx U_{\infty} B [\sigma_{i\infty} (1 - K_i) (a_s - 2\Delta_u) + 2\Delta_u \int_0^{\xi_u} \sigma_i(\xi_u) \xi_u^n d\xi_u - 2\Delta_u K_i \int_0^{\xi_u} \sigma_i(\xi_u) d\xi_u] \quad (18)$$

Current  $J_i$  is determined from formula (18) (at assigned  $K_i$ ) by numerical integration taking into account expressions (6)-(8).

<sup>1</sup>According to Fig. 1 current into load is determined by the following integral:

$$J_H = 2 \int_0^{a/2+b} j_y(z) dz.$$

### § 3. THE EVALUATION OF THE INFLUENCE OF VARIOUS FACTORS ON THE REAL RESISTANCE OF THE BOUNDARY LAYER

The calculation of near-electrode resistance represents a complex problem and the solutions found relate only to special cases [17, 20, 21, 23]. In the wall areas of the boundary layer on the electrode the electron concentration and conductivity can be decreased strongly, which leads to the appearance of diffusion flows to the electrode and an increase in Joule heat release. In this case it is necessary to solve jointly the equations of diffusion and energy for electrons [20, 23]. At distances from the electrode of the order of Debye radius the quasineutrality of the plasma is disrupted and the potential distribution is determined by the Poisson equation. Let us note that for typical operating conditions of equilibrium MHD generators the Debye radius at  $T_e \sim 1500^\circ\text{K}$  is much greater than the mean free path of electrons.

In engineering practice for the determination of near-electrode resistance they use various simplified method, the applicability of which under those or other conditions is not frequently founded. During the analysis of near-electrode resistance usually two components are separated out, one of which is specified by the boundary layer, and the other - by contact resistance of the "plasma-electrode" [9, 14]. The computation of the resistance of the boundary layer is connected with the integration of specific resistance for the boundary layer. In this case in a number of works the lower limit of integration is assumed equal to zero, the profile of reduced temperature is usually taken as corresponding to the exponential velocity profile with an



index equal to 1.7, and the influence of viscous sublayer and changes of Hall's parameter in boundary layer are not considered [11, 14]. Such an approach can lead to significant errors, especially in the case of "cold" electrodes. In works [9, 26] on the basis of the comparison of calculation with experiment (the conclusions of the works [21] were used) it is recommended to take a lower limit during integration over the boundary layer thickness equal to the Debye radius. In this case the resistance of the boundary layer should not depend on the density of current.

Let us examine the influence of various factors on the magnitude of the real resistance of a turbulent boundary layer.

Let us present the near-electrode resistance in the form of the sum of two in series connected resistances:

$$R_3 = R_{31} + R_{32},$$

where  $R_{31}$  - the resistance of boundary layer on an electrode with thickness  $(\Delta_3 - z_3)$ ;  $R_{32}$  - contact resistance of the "plasma electrode," to which the jump in potential  $\Delta V_3$  corresponds. In this work the calculation of  $R_{32}$  was not conducted, but the potential jumps on the cathode  $\Delta V_{c2}$  and anode  $\Delta V_{a2}$  were considered known. The values of  $\Delta V_{c2}$  and  $\Delta V_{a2}$  can be calculated during the clarification of the detailed mechanism for processes near the surface of the electrode or determined from experimental findings.

If we disregard the boundary layer effect on the insulator on near-electrode resistance, then  $R_{31}$  can be expressed in the following form:<sup>1</sup>

$$R_{31} = \frac{\Delta_3}{\alpha - 2\Delta_0} \rho_{31} \approx \frac{\Delta_3}{\alpha} \rho_{31}, \quad (19)$$

---

<sup>1</sup>Here the distance along axis "y" is counted off from the surface of the electrode.

$$\rho_{31} = \int_{\tau_0/\Delta_3}^1 \sigma^{-1}(\xi_3) d\xi_3, \quad (20)$$

where  $\xi_3 = y/\Delta_3$ .

Some authors assume [9, 21] that the value of  $\tau_0$  is determined by the characteristic value of the order of the length of Debye shielding in plasma  $\tau_D$ :

$$\tau_D = \sqrt{T(\tau_D)/4\pi e^2 n_e(\tau_D)}. \quad (21)$$

However, already at  $y > \tau_0$  even in molecular gases there is the possibility of a significant breakaway of the temperature of electrons  $T_e$  from the temperature of heavy particles under the effect of the electric field increasing toward the electrode ( $E^*(y) = j_y/\sigma(y)$  in the model in question  $j_y = j_{y\infty} = \text{const}$ ). We evaluate the influence of these approximations on near-electrode resistance  $\rho_{31}$  and the conditions under which the model approximation  $\tau_0 = \tau_D$  becomes unacceptable due to the appearance of a noticeable breakaway of the temperature of electrons.

3.1. Let us examine the case  $\tau_0 = \tau_D$  (we consider that  $\tau_0 \ll \tau_D$ ). Usually  $\tau_0 \ll \delta_A$  and it is possible to isolate the real resistance of the turbulent nucleus and viscous sublayer:

$$\rho_{31} = \rho_\tau + \rho_A. \quad (22)$$

The results of calculations  $\rho_{31}$  for the typical combustion products of hydrocarbon fuel [3, 22] ( $\gamma_{\text{sp}} = 1.12$ ;  $M_\infty = 1.5$ ;  $T_\infty = 3000^\circ\text{K}$ ;  $\beta \ll 1$ ) for  $T_s = 300^\circ\text{K}$ - $2100^\circ\text{K}$  and  $n = 1/5$ - $1/10$  are presented in Figs.

5, 6. At  $n \approx 1/7$  resistance  $\rho_{31}$  decreases by an order with an increase of  $T_s$  from  $300^\circ\text{K}$  to  $1800^\circ\text{K}$  (Fig. 5). The small change in the exponent  $n$  at  $T_s \leq 1000^\circ\text{K}$  exerts a noticeable influence on the resistance of the boundary layer.

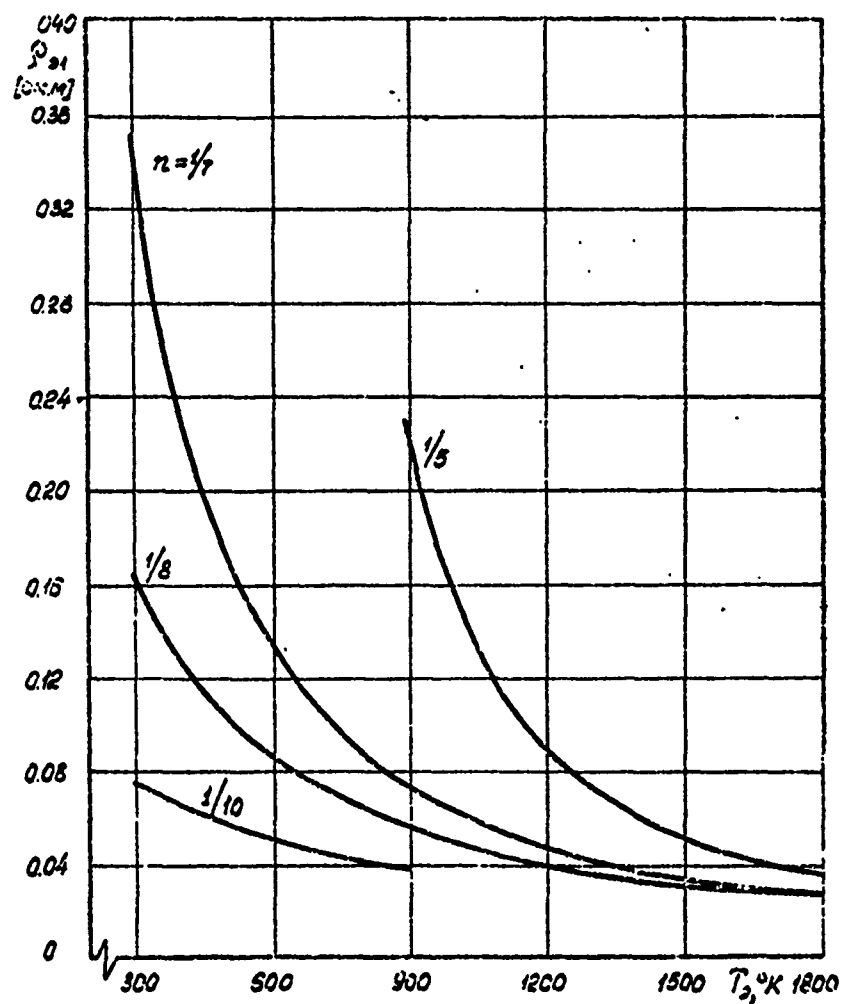


Fig. 5. The dependence of the resistance of the boundary layer on the temperature of an electrode ( $l \sim l''$ ).

Designation: [ohm.m = ohm.m].

Figure 6 shows the influence of the viscous sublayer on  $\rho_s$ , while maintaining a similitude of profiles of the given velocity and temperature for  $n = 1/7$  in the nucleus. It is evident that simple exponential approximation of the velocity profile up to the wall, usually utilized for computation [9, 14], at low temperatures can lead to an understating of the magnitude of  $\rho_s$ , by several times in comparison with the two-layered arrangement of the boundary layer even at small  $\delta_\lambda$  ( $\delta_\lambda/\Delta_s \sim 0.005$ ).

3.2. The distance from the electrode at which breakaway  $T_e$  becomes significant can be evaluated by the equation of energy balance of the electron gas:

$$\sigma E^* = \frac{3}{2} (T_e - T) n_e \sum_k 2 \delta_k \frac{m_e}{M_k} \langle v_{ek} \rangle. \quad (23)$$

If the effective values of the collision rate  $\nu_e^*$ , the coefficient of inelastic losses  $\delta_{\text{in}}$  and the atomic masses (of the molecule) are known, then the electric field in which the assigned breakaway of the temperature of electrons ( $T_e - T$ ) appears is calculated as:

$$E^* = \text{const} \times \nu_e^* \sqrt{\delta_{\text{in}} (T_e - T)}. \quad (24)$$

Expression (24) makes it possible in the case of known dependence  $\sigma(y)$  to  $j_y = j_{y\infty}$  determine the characteristic distance  $z_\epsilon$  from the electrode at which at a distance  $y = z_\epsilon$  the "heating" field  $E^* = j_{y\infty}/\sigma(y)$  will guarantee the assigned breakaway of the temperature of electrons. During the calculation of the breakaway of electron temperature it is assumed that the diffusion length of the ions  $\ell_i$  [24] is less than the value  $z_\epsilon$ . Figures 7, 8, and 9 depicts the comparative calculations for a two-layered arrangement of the boundary layer at  $z_s = z_\epsilon$  and  $z_s = z_\epsilon$  at  $(T_e - T) \delta_{\text{in}} = 10^4$  °K and various current densities.

At  $z_s = z_\epsilon$  ( $z_\epsilon \neq z_0$ ) the value of  $z_\epsilon$  ( $j$ ,  $T_w$  - assigned) is determined by the conductivity  $\sigma(z_\epsilon)$ , which is attained at an identical temperature at different distances from wall, which gives rise to a weak dependence of  $\rho_s$  on the structure of boundary layer.

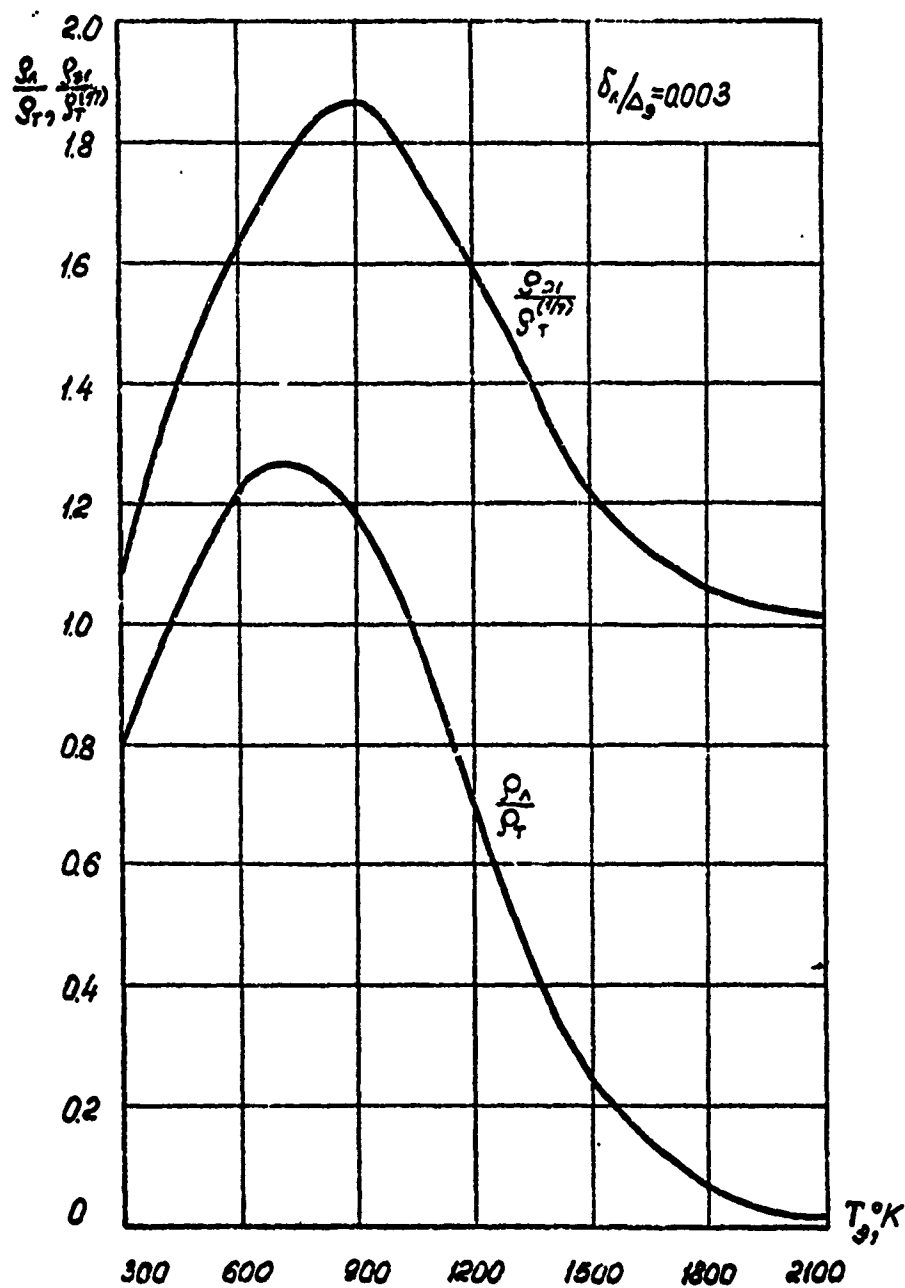


Fig. 6. The influence of the viscous sublayer on the resistance of the boundary layer ( $\delta_\Delta/\delta = 0.003$ ).

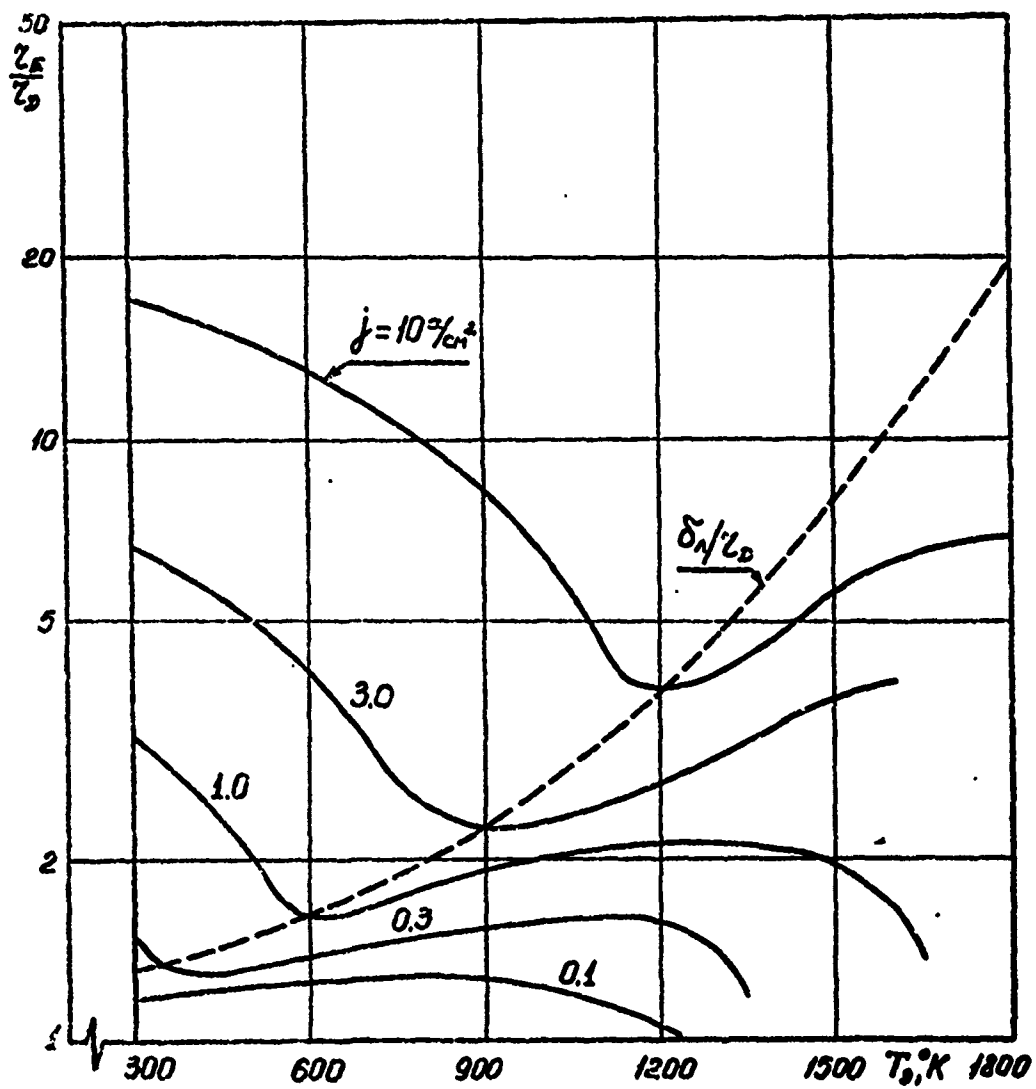


Fig. 7. Dependence of  $\gamma_s/\gamma_0$  on temperature of the electrode.

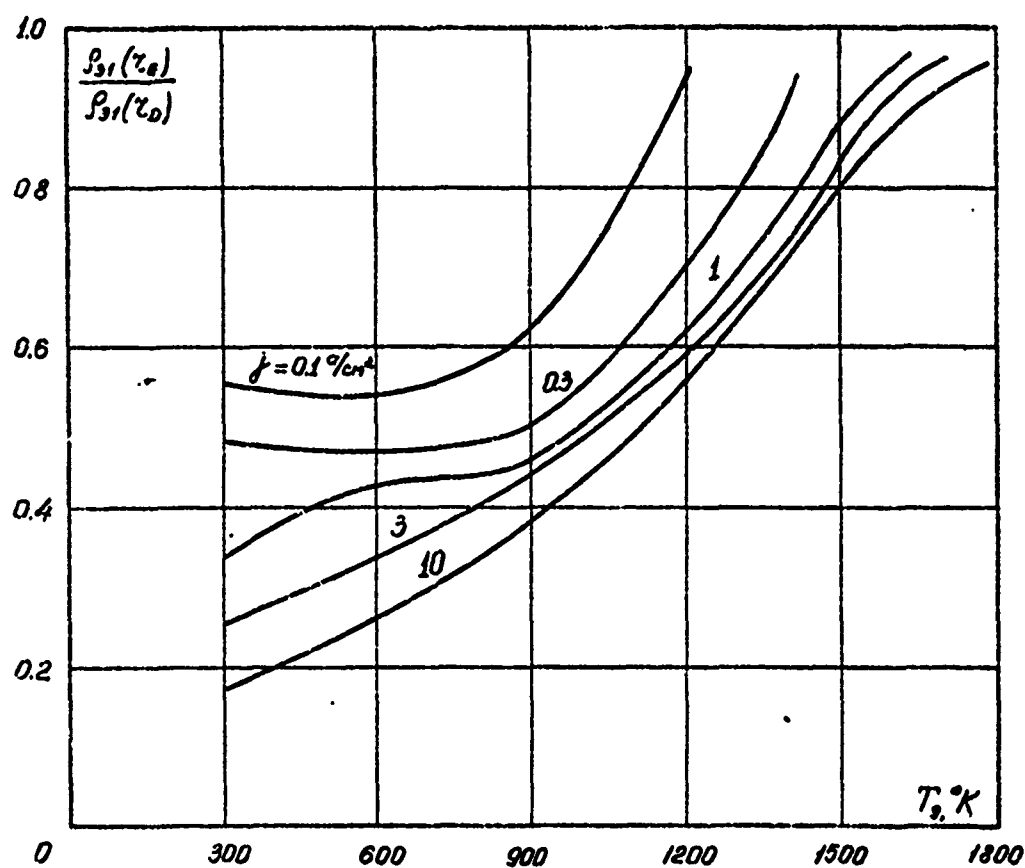


Fig. 8. Relationship of the resistances of the boundary layer at  $z_s = z_0$  and  $z_s = z_0$ , depending on the temperature of the electrode.

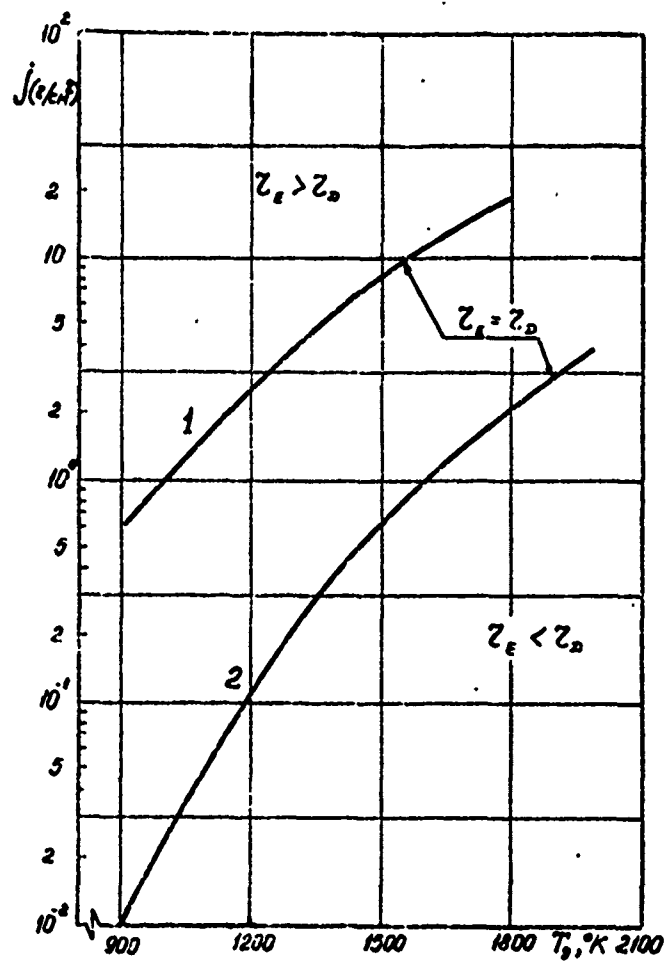


Fig. 9. The projection of the line of intersection of surfaces  $z_e = f_e(j, T_e)$  and  $z_o = f_o(j, T_o)$  on the plane  $j, T_o$  (curve 1 - for the exponential velocity profile, curve 2 - for the two-layered circuit).



The influence of the temperature profile on the magnitude of  $\rho_{31}$  is similar to the case  $z_s = z_D$ .

Therefore during the calculation of  $\rho_{31}$  one ought to select as the lower limit of integration in expression (20) the maximum value from  $z_D$  and  $z_s$ . Since the influence of the resistance of the boundary layer on the characteristics of the generator (see also § 5) is especially significant only at  $T_s \leq 1500^\circ\text{K}$ , then in specific interesting cases ( $p \leq 10$  ata;  $j \approx 1-5$  a/cm<sup>2</sup>;  $\delta_{\text{app}} \approx 20-50$ ;  $M_{\text{app}} \approx 30$ )  $z_s > z_D$  (Fig. 9). Let us note that at  $z_s = z_s$  the value of  $\rho_{31}$  drops with an increase of current density, other conditions being equal. However, this does not mean that the complete near-electrode resistance  $\rho_3$  decreases, i.e., resistance  $\rho_{32}$  also depends on the current density.

The analysis conducted shows that the existing methods for the calculation of the real resistance of the boundary layer, which are based on a break in integration at a certain arbitrary distance from the electrode  $z_s$ , give results which are strongly dependent on the selection of this boundary. Integration in formula (20) up to maximum value from  $z_D$  and  $z_s$  apparently gives an evaluation of the minimum value of the near-electrode resistance. Therefore the performance calculation of an MHD generator is advantageously conducted for a series of the assigned (parametric analysis) or experimental values of the equivalent resistances of the circuit of substitution. Specifically parametric analysis will make it possible to determine the permissible values of internal resistance of the MHD channel.

#### § 4. THE INFLUENCE OF NEAR-ELECTRODE RESISTANCE ON THE EFFECTIVE TRANSVERSE CONDUCTIVITY OF A FARADAY GENERATOR

It is known that in real MHD channels with continuous electrodes there are always both significant near-electrode resistance and two-dimensional effects in the distribution of potential and current, especially when the Hall effect exists. These effects can lead to the appearance of a longitudinal (Hall) component of electric field and, consequently, to an increase in effective transverse conductivity in the flow core. The influence of the two-dimensional nature on the characteristics of generators with a short channel (number of gages  $K=L/h \leq 3$ ) was examined in a number of works [25, 4, 6]. However, in this case the influence of the heterogeneities of flow connected with the presence of boundary layers was not analyzed. Near-electrode resistances prevent the complete closing of Hall current in a generator with continuous electrodes and thereby lead to the appearance of a longitudinal electric field and, consequently, to an increase in transverse conductivity in the flow core as compared with the case of uniform distribution.

We will evaluate the influence of this effect on the characteristics of an MHD channel with continuous electrodes.

Let us write the components of current densities in the following form:

$$j_y = \sigma_e E_y^* (1 + \beta \frac{E_z}{E_y^*}), \quad (25)$$

$$\tilde{j}_x = \sigma_1 (E_x - \beta E_y^*), \quad (26)$$

where

$$E_y^* = -\mu_\infty B + E_y.$$

Let us assume that Hall current flows on the electrodes in sections with a characteristic length  $l_2$  and  $l_1$  on the ends (Fig. 10). Then with a weak change in parameters along the length of the channel the bond between effective Hall current  $\tilde{j}_x$  and field  $\tilde{E}_x$  can be written as (for simplicity we will consider  $a = 1 = \text{const}$ ,  $h = \text{const}$ ):

$$\tilde{E}_x L^* \approx \tilde{j}_x \frac{h}{c} \left( \rho \Delta + \sigma_\kappa^{-1} \frac{h}{\alpha_\kappa} \right), \quad (27)$$

where

$$\frac{1}{\rho} \left( \rho \Delta + \sigma_\kappa^{-1} \frac{h}{\alpha_\kappa} \right) = \frac{1}{2} \left[ \left( \rho_1 \frac{\Delta_1}{l_1} + \rho_2 \frac{\Delta_2}{l_2} \right) + \frac{h}{\alpha_\kappa} \left( \frac{1}{\sigma_{12} l_1} + \frac{1}{\sigma_{12} l_2} \right) \right],$$

$\rho_{1,2} = (\rho_{31} + \rho_{32})_{1,2}$  ( $\rho_{31}$  and  $\rho_{32}$  have been determined above),  $\alpha_\kappa$  - numerical coefficient,  $l_1' \sim l_1$ ,  $l_2' \sim l_2$ . The second term in equation (27) appears due to the end spreading of current.

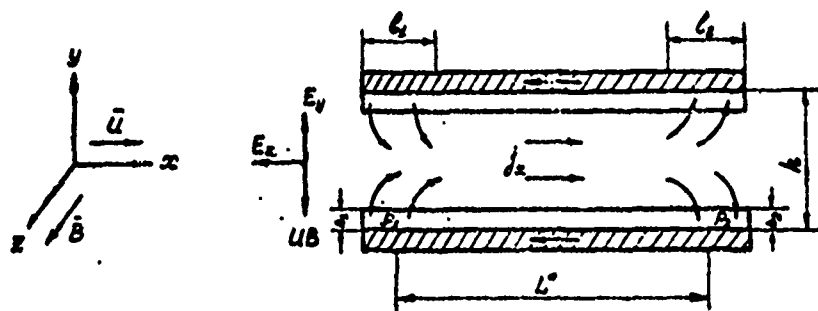


Fig. 10. The circuit of closing of the Hall current in a channel with continuous electrodes.

Combining expressions (26) and (27), we obtain for relation  $\frac{\tilde{E}_x}{E_y^*}$  the following formula:

$$\frac{\tilde{E}_x}{E_y^*} = \beta \left[ 1 + \frac{\ell L^*}{\sigma_i h (\rho_{\Delta} + \sigma_i^{-1} h / \alpha_{\kappa})} \right]^{-1}. \quad (28)$$

The expression for the density of transverse current we present in the form of

$$j_y = U_{\infty} B (1 - K_{\infty}) \sigma_i \left[ 1 + \frac{\beta^2}{1 + \frac{\ell L^*}{\sigma_i (\rho_{\Delta} + \sigma_i^{-1} h / \alpha_{\kappa})}} \right]. \quad (29)$$

This expression can be reduced to the common form:

$$j_y = \sigma_{\text{eff}} U_{\infty} B (1 - K_{\infty}), \quad (30)$$

where

$$\sigma_{\text{eff}} = \sigma_i \left( 1 + \frac{\beta^2}{1 + \alpha_{\kappa}^2 \ell / h K} \right)^{-1} \quad (31)$$

the effective transverse conductivity of the channel,

$$\alpha^* = \frac{h \sigma_i^{-1}}{\rho_{\Delta} + \sigma_i^{-1} h / \alpha_{\kappa}} = (\alpha_{\kappa}^{-1} + \alpha^{-1})^{-1}, \quad \alpha = \frac{h \sigma_i^{-1}}{\rho_{\Delta}}. \quad (32)$$

The appearance of a longitudinal electric field leads to an increase in effective conductivity as compared with its value from the one-dimensional theory  $\sigma_i$  for value  $\left( 1 + \frac{\beta^2}{1 + \alpha_{\kappa}^2 \ell / h K} \right)$ . Since near-electrode resistances lead, on the one hand, to obvious added losses in total generated power, and on the other (as also common end effects [25, 4, 6]) - to an increase in effective conductivity in the flow core, then under specific conditions there is the possibility of existence of optimum in the dependence of the complete generated power on the characteristics of the near-electrode effects.

Output power is maximum under the condition of agreement of resistances of the channel and load:

$$R_H = R_{i\text{эфф}} + 2\rho \frac{\Delta}{\alpha h},$$

where

$$R_{i\text{эфф}} = h / \sigma_{\text{эфф}} \alpha L^*.$$

With a small change in conductivity and velocity in the section of the electrode with the length  $L^*$  the expression for maximum power takes the following form:

$$P = \frac{1}{4} (UBh)^2 (R_{i\text{эфф}} + 2\rho \frac{\Delta}{\alpha L^*})^{-1} \quad (33)$$

Let us rewrite this formula, using the correlation

$$P = P_0 \psi(\alpha, \alpha_\kappa, \beta, \frac{l}{h} \kappa), \quad (34)$$

where

$$P_0 = \frac{1}{4} \sigma_0 U^2 B^2 \alpha h L^*,$$

$$\psi = (1 + \beta^2)^{-1} \frac{\alpha^2 (1 + \beta^2 + \alpha_\kappa \frac{l}{h} \kappa) + \alpha \alpha_\kappa (1 + \beta^2)}{\alpha^2 (1 + \alpha_\kappa \frac{l}{h} \kappa) + \alpha [2(1 + \beta^2) + \alpha_\kappa + 2\alpha_\kappa \frac{l}{h} \kappa] + 2\alpha_\kappa (1 + \beta^2)}. \quad (35)$$

The condition of the extreme of function  $\psi$  on parameter  $\alpha$  ( $\partial\psi/\partial\alpha = 0$ ) leads to the following equation:

$$\alpha^2 [2(1 + \beta^2 + \alpha_\kappa \frac{l}{h} \kappa)^2 - \alpha_\kappa^2 \frac{l}{h} \kappa \beta^2] + 4\alpha \alpha_\kappa (1 + \beta^2) \times \\ (1 + \beta^2 + \alpha_\kappa \frac{l}{h} \kappa) + 2\alpha_\kappa^2 (1 + \beta^2)^2 = 0. \quad (36)$$

Analysis shows that this equation has a positive root in the case of fulfillment of the following conditions:

$$\alpha_K > \frac{1+\beta^2}{\beta \sqrt{\frac{l}{2h} K - \frac{l}{h} K}} , \quad \beta^2 > 2 \frac{l}{h} K. \quad (37)$$

The values of parameters  $\alpha_K$  and  $l$  can be evaluated by means of a comparison with the results of two-dimensional calculation. The computation of power using formula (34) leads to satisfactory agreement with the results of work [4] at  $l/h \approx \beta/4$  for  $\beta = 0-10$ . In this case inequalities (37) are fulfilled at

$$\alpha_K > 8(1+\beta^2)\beta^2, \quad \beta > K/2, \quad (38)$$

i.e., for  $K \geq 3$  at  $\beta \geq 2$ ,  $\alpha_K \geq 8$ .

Figure 11 shows the dependence of relative power  $\varphi = P/\mu_0$  from  $\beta$  at  $l = \beta h/4$ ;  $K=5$ ;  $\alpha=2$ ;  $4$ . Curve 1 corresponds to calculations in the one-dimensional theory ( $\sigma_{eff} = \sigma_s$ ,  $\alpha_K = \infty$ ,  $P \sim 1/(1+\beta^2)$ ); curve 2 - using formulas (35), (36) at  $\alpha_K = \infty$ ; curve 3 - at  $\alpha_K = 2$ . It is evident from this figure that at large  $\beta$  the difference from the one-dimensional theory is considerable and at  $\beta = 1-2$  the increase in output power as compared with one-dimensional calculation comprises  $\sim 1.5$ .

Figure 12 depicts the dependence of  $\varphi \times (1+\beta^2)$  on  $\alpha$  at  $h = 5$ ;  $l = \beta h/4$ ;  $\beta = 1$ ;  $5$  for cases which correspond to the curves in Fig. 11. At small  $\alpha$  (large  $\beta$ ) function  $\varphi \times (1+\beta^2)$  depends weakly on  $\alpha_K$  and it is possible to assume  $\alpha_K = \infty$ . At  $\alpha \rightarrow \infty$  function  $\varphi \times (1+\beta^2)$  approaches the limiting value

$$[(1+\beta^2)P/\mu_0]_{\infty} = 1 + \frac{\beta^2}{1+\beta \alpha_K K/4}.$$

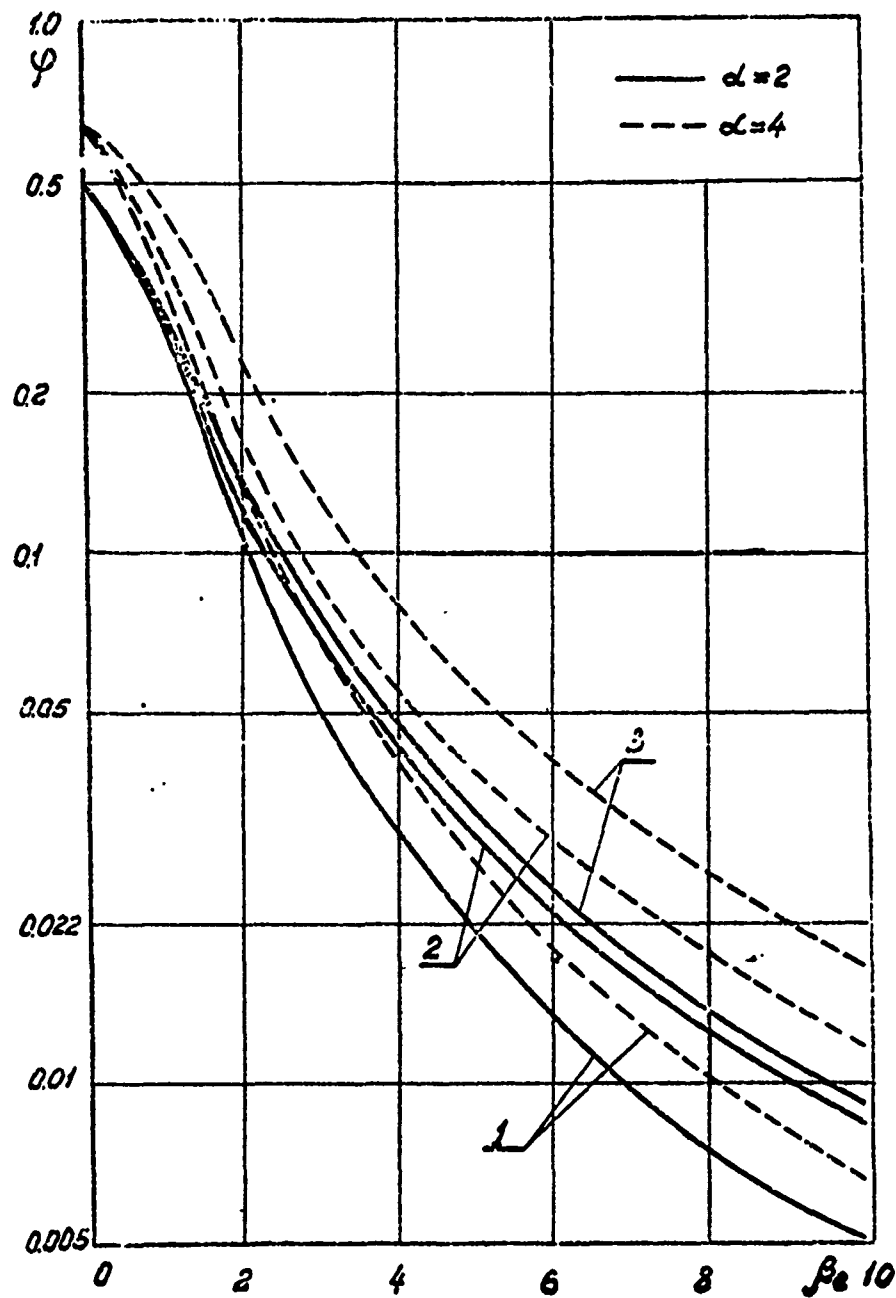


Fig. 11. The dependence of relative power  $\varphi = p/p_i$  on the Hall parameter at  $k = 5$ ;  $\alpha = 2$ ; 4;  $\ell = \beta h / \lambda$  (1 - one-dimensional theory; 2 -  $\alpha_x = \infty$ ; 3 -  $\alpha_x = 2$ ).

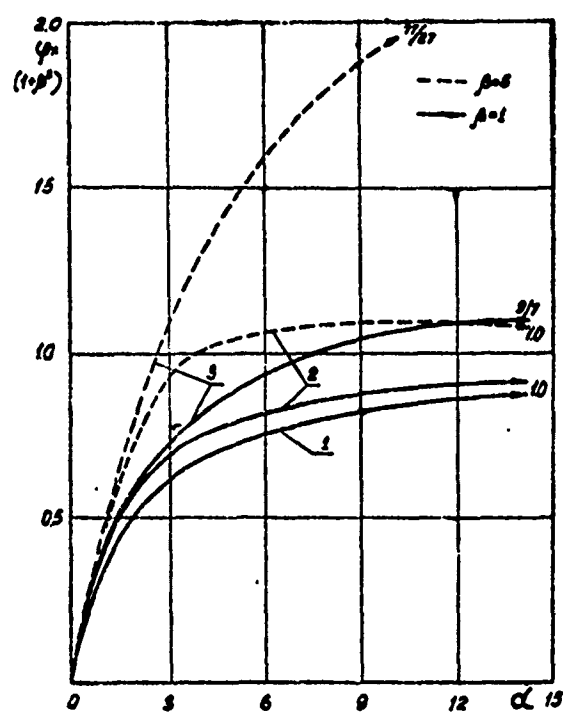


Fig. 12. The dependence of relative power  $\varphi/(1+\beta^2)$  on  $d$  at  $k=5$ ;  $\beta = 1; 5$ ;  $\varphi = \beta h/4$  (1 - one-dimensional theory; 2 -  $d_n = \infty$ ; 3 -  $d_n = 2$ ).



### § 5. THE INFLUENCE OF HETEROGENEITIES OF FLOW ON THE CHARACTERISTICS OF AN MHD GENERATOR WITH A SMALL-SCALE CHANNEL

As an example let us examine the electrical characteristics of an MHD generator channel on the typical combustion products of hydrocarbon fuel [2, 3, 22]. Let the area of flow cross-section be  $\sim 10 \text{ cm}^2$ , and deposits on the walls of the channel are absent. The relative sizes of the channel and the boundary layers are the following:

$$\alpha=1; h=3.5; \delta_s=\delta_u=0.34; \Delta_s=\Delta_u=0.37; \delta_h=2 \cdot 10^{-3}.$$

Let us select for evaluation the following parameters of gases:

$$\gamma=1.12; M=2.2; T_\infty=2500^\circ\text{K}; P=2 \text{ [atm (abs.)]}; \beta_\infty=0.5; \\ \rho_r=0.88; \mu=5.5 \cdot 10^{-5} \text{ kg/ms } n=n_r=1/7.$$

The induction of magnetic field  $B = 3 \text{ ml}$ . The temperature of the walls:

- a) "cold" walls -  $T_s = 1000^\circ\text{K}; T_u = 1500^\circ\text{K};$
- b) "hot" walls -  $T_s = 2000^\circ\text{K}; T_u = 2300^\circ\text{K}.$

Figure 13 shows the current-voltage characteristics for a unit of length of channel at  $\Delta V_{\alpha_2} = \Delta V_{\alpha_1} = 0$ , in relative coordinates  $V = v/u_\infty B h$ ,  $\tilde{j} = j/\sigma_\infty u_\infty B a$  at  $\chi_s = \chi_n$  and  $\chi_s = \chi_k$  (for  $\delta_{\text{app}}(T_e - T) = 10^4^\circ\text{K}; j_y = 1; 10\% \lambda$ ), and in Fig. 14 - the dependence of dimensionless output power  $\tilde{N} = N/\sigma_\infty u_\infty^2 B^2 a h$  on current in the load, calculated from the

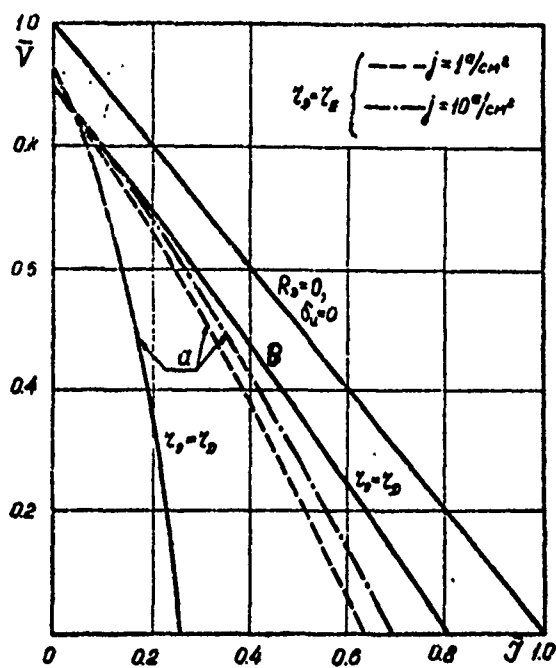
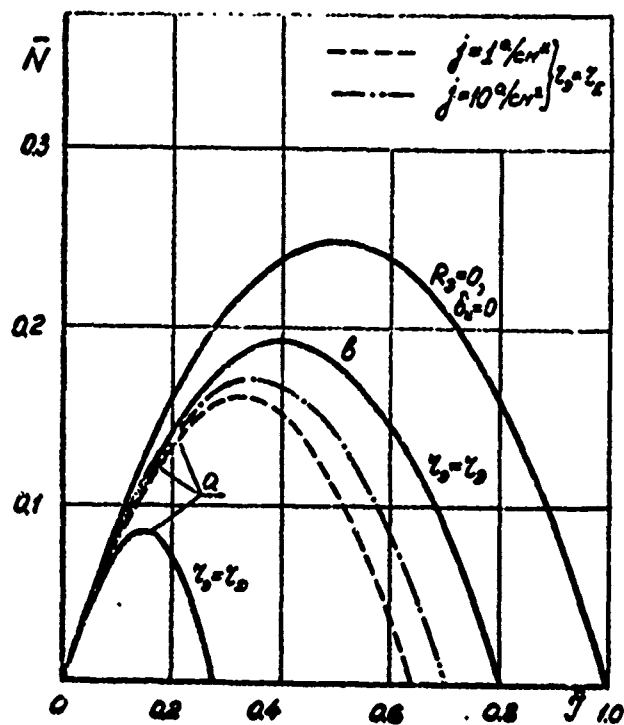


Fig. 13. Volt-ampere characteristics of a model MHD channel ( $\alpha - T_s = 1000^\circ\text{K}$ ,  $T_u = 1500^\circ\text{K}$ ;  $\beta - T_s = 2000^\circ\text{K}$ ,  $T_u = 2300^\circ\text{K}$ ).

Fig. 14. The dependence of output power

$\bar{N} = \frac{N}{\sigma_a u_a \beta \alpha h}$  on current in the load  $\bar{J} = \frac{J}{\sigma_a u_a \beta \alpha}$  ( $\alpha - T_s = 1000^\circ\text{K}$ ,  $T_u = 1500^\circ\text{K}$ ;  $\beta - T_s = 2000^\circ\text{K}$ ,  $T_u = 2300^\circ\text{K}$ ).



one-dimensional method with the utilization of a two-layered circuit of boundary layer. Figure 15 depicts the dependence of the short-circuiting current  $\bar{J}_{K,s} = J_{K,s} / \sigma_m U_m B a$  on the temperature of insulating wall at  $\tau_s = \tau_0$  and  $T_s = 1000^\circ\text{K}$ ,  $1500^\circ\text{K}$ ,  $2000^\circ\text{K}$ .

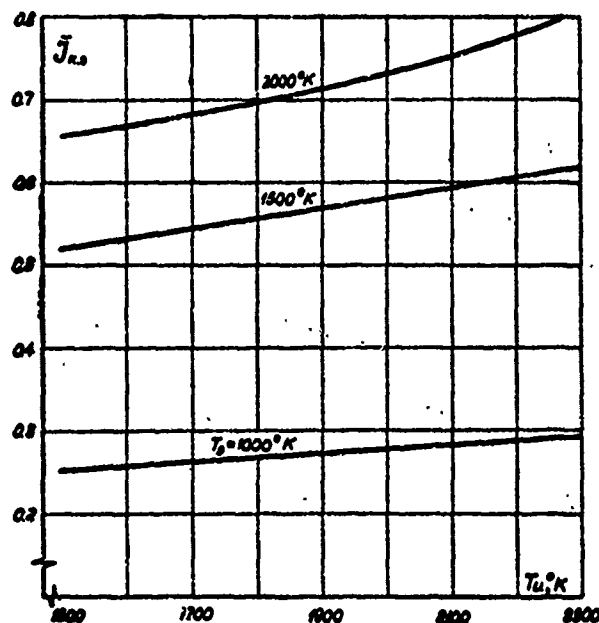


Fig. 15. The temperature effect of insulating wall on the short-circuiting current

$$\bar{J}_{K,s} = \frac{J_{K,s}}{\sigma_m U_m B a} (\tau_s = \tau_0).$$

The calculations conducted showed that with "cold" channel walls the output power of the generator can be 2-3 times less than its value not allowing for the boundary layer. The method of calculation of near-electrode resistance exerts a strong influence on the calculated value of near-electrode resistance and therefore output power at  $T_s \sim 1000^\circ\text{K}$  (Fig. 14). Since in the given calculation the component of near-electrode resistance  $\rho_{s2}$  was not considered, then the results obtained bear a semiquantitative nature. The open-circuit voltage due to leakages in the turbulent boundary layers decreases only by  $\sim 10\%$ , and the short-circuiting current falls by  $\sim 15\%$  with the reduction in the temperature of the insulating walls from  $2000^\circ\text{K}$  to  $1000^\circ\text{K}$ .

## CONCLUSION

A local circuit of substitution for an MHD generator channel has been proposed which makes it possible in a gas-dynamic approximation to calculate the influence of the heterogeneities of flow on the electrical characteristics of the generator.

An approximate computation of the basic elements of the substitution circuit has been conducted. It has been shown that in the nominal regimens of operation of an MHD generator it is possible to practically disregard the leakages of current in turbulent boundary layers on the insulating walls. In this case the circuit of substitution (Fig. 3) is considerably simplified and the basic influence on the characteristics of the generator will be exerted only by near-electrode effects, especially the resistance of thermal boundary layers, and also leakage on the material of the insulating walls and the layer of electro-conductive deposition (if they are significant).

At low electrode temperatures ( $T_e \leq 1000^\circ\text{K}$ ) the selection of the characteristic integration limit of  $\chi_e$  exerts a powerful influence on the magnitude of the components of resistance of the boundary layer  $R_{e1}(\rho_{e1})$  during calculation in the two-layered circuit of boundary layer, and condition  $\chi_e = 0$  can lead to an increase of  $R_{e1}$  by an order. However, arbitrariness in the selection of  $\chi_e$  at  $\chi_e > \chi_0$  (for instance, the condition  $\chi_e = \chi_0$  at the assigned parameter  $\sqrt{\delta_{ep}(T_e - T)}$ ) in the case  $T_e \gtrsim 1000^\circ\text{K}$  weakly influences the result of calculation of  $R_{e1}(\rho_{e1})$ , especially with the exponential

approximation of the velocity (temperature) profile up to the electrode, and at  $T_e \approx 2000^\circ\text{K}$  it is possible to assume  $\chi_e = 0$ . This apparently can explain the satisfactory agreement of the results of the calculation of the resistance of the thermal boundary layer at  $\chi_e = \chi_D$  with the experimental values obtained in work [9].

The resistance of the thermal boundary layer on the electrode at an assigned thickness is determined mainly by the temperature of the inner surface of the electrode wall, especially at  $\chi_e = \chi_D$ .

The structure of the boundary layer and exponent  $n$  in the case of exponential approximation of the profiles of velocity and temperature exert a noticeable influence on the real resistance of the boundary layer.

At the large current densities the profiles of velocity and temperature can differ noticeably from those accepted in calculation and, furthermore, a breakaway of electron temperature and the emergence of a micro-arc mode of the flow of current are possible.

Thus the correct calculation of near-electrode resistance requires the careful analysis of processes which accompany the flow of current in the interelectrode interval of the channel. Two-dimensional effects and near-electrode resistance in the channel of a Faraday generator with continuous electrodes at  $Re \gg 1$  can cause a noticeable longitudinal electric field, which leads to an increase in transverse conductivity in the interelectrode interval of the channel. Evaluations show that the increase in output power as compared with one-dimensional calculation can be significant ( $\sim 1.5$  times at  $\beta \approx 1-2$ ,  $K \approx 5$ ).

The concrete calculation of a small-scale generator showed that at an electrode temperature of  $\sim 1000^\circ\text{K}$  output power can be 2-3 times less than its value not allowing for the resistance of the boundary layer.

The boundary layer effect can be significant also in heavy installations at large values of parameters  $L_e/\delta$ . If far from the separation point for a hydraulically smooth wall  $\Delta \approx \delta \sim L_e^{1/2}$ , then at

an identical number of gages  $L_K/h = \text{const}$  and an identical temperature rate  $\frac{R_i}{R_e} \sim \frac{\Delta}{h} \sim \frac{K}{L_K^{1/(1-m)}}$ . Since usually  $m \approx 0.8$ , then the correlation between the thickness of the boundary layer and the transverse dimension of the channel depends weakly on its length. Therefore, for the reduction of relative resistance in the boundary layer (or the loss of voltage in the channel) other conditions being equal it is necessary to select a relatively small number of gages.

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